PROBLEM OF THE MONTH

POM

2015-2016

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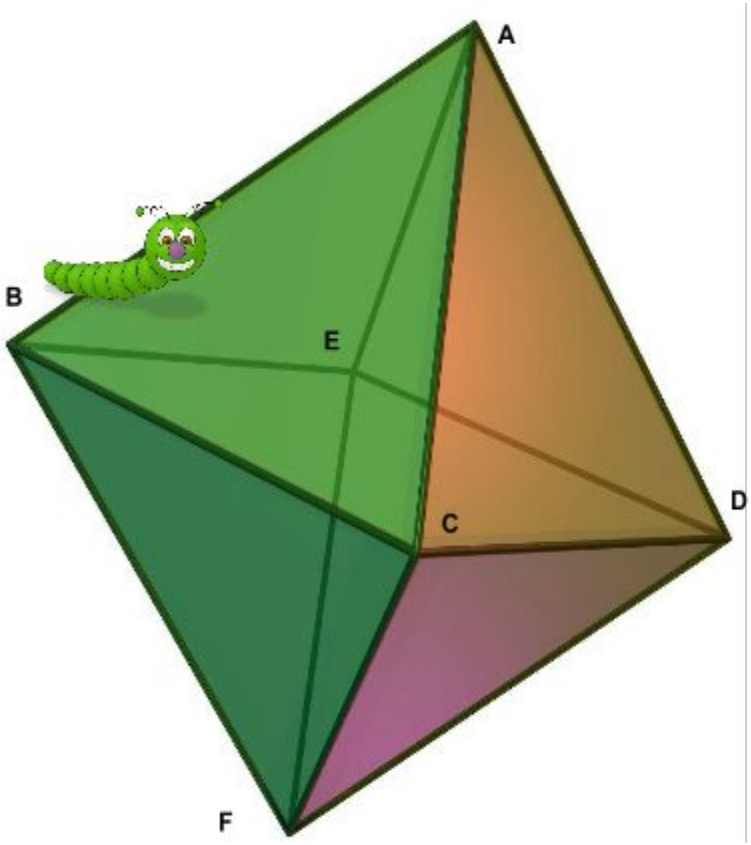
MATHEMATICS

5 points:

Each face of the octahedron ABCDEF in the Figure is an equilateral triangle with side ***a***. Caterpillar named Plato originally sits in the center of the face ABC, and wants to move to the center of face CDF. Plato can only move along the octahedron surface. What is the shortest distance that the Caterpillar has to travel to the destination?



Hint: Imagine that you want to fold the octahedron out of paper. In order to do this, you would have to cut out a special shape. Try to construct this shape, and identify on your picture the points between which the Plato has to travel.

10 points:

Each face of the elongated octahedron ABCDEF in the Figure is an isosceles triangle with base ***b*** and sides ***a*** (for instance, **AB=AC=a** and **BC=b**). Caterpillar named Plato originally sits at the vertex B and wants to move to the opposite vertex D (BD is the diagonal of the square BCDE). Plato can only move on the octahedron surface. What is the shortest distance that the Caterpillar has to travel to the destination?

Hint: Imagine that you want to fold the octahedron out of paper. In order to do this, you would have to cut out a special shape. Try to construct this shape, and identify on your picture the points between which the Plato has to travel.

PHYSICS

5 points:

Two wooden blocks of the same mass are glued together. The composite block is floating in water. What part of it is submerged into water if the densities of blocks are 500 kg/m3 and 1100 kg/m3, respectively?

Hint: Try to equate the Archimedean buoyancy force to the total weight of the composite block.

10 points:

A conical vessel has a small hole at the bottom (at the vertex of the cone). It is known that when V liters of water is placed into the vessel the velocity of fluid exiting the hole is U. What will be the velocity if one fills the vessel with $2V$ liters of water?

Hint: The exit velocity of the fluid from a small hole can be found in the following way. The hydrostatic pressure at depth h is equal to $P = \rho g h$, where ρ is the density of the fluid, and h is the height of the water level in the vessel. You can use Bernoulli Principle to relate this pressure *P* to the speed of flow v that it generates: $P = \rho v^2/2$.

CHEMISTRY

5 points:

Imagine you are playing the Escape the Room game. You found that to open a final lock you need to apply a weak electric signal to two wires connected to the doors. That will trigger the electronic device that controls the lock. The voltage of this signal should be 1 to 5 V. In the room, you found the following list of items:

1. A flashlight with a dead battery
2. A glass bowl with fruits (3 oranges, 2 lemons, 5 apples, grape, kiwi)
3 A rope
4 Scissors
5 A box with screws, zinc coated nails, stainless nuts and bolts
6 A glass of water
7 A wallet with 5 one dollar bills, and various coins (mostly dimes and pennies)
8 A bottle of liquid soap, a brush and toothpaste
9 A roll of copper wire
10 A bottle of Sprite

Which above listed items can you use to escape the room, and how concretely will you do that?

Hint: To escape the room, you need to make a battery. Try to google what the galvanic cell is, and which metals should be used to get needed voltage.

10 points:

A group of students, supervised by their teacher, Alice, and a technician, Bob, did some experiments in the school lab with the following solutions.

$LiNO_3$, $Pb(ClO_4)_2$, Na_2S, $Mg(ClO_4)_2$, $Ba(NO_3)_2$

After these experiments, Bob collected the remaining solutions into two waste bottles (some solutions went to one bottle, the bottle **A**, whereas other solution were poured to the another bottle, the bottle **B**. Since lead and barium are toxic heavy metals, he decided to put the “Hazardous waste! Heavy metals!” label on both bottles. However, Alice tell him to wait a little bit. “Look”, she said, “The solutions in both bottles are transparent and colorless. I think that means only one bottle contains heavy metals, whereas another bottle is a non-heavy metal waste. Let's check that”. She took a small

amount of liquid from each bottle and mixed them in a test tube. A black precipitate formed immediately. “Ok, that is what I expected”, Alice said. “Let's make additional tests”. She poured a small amount of the waste solution **A** into a clean test tube and the same amount of the solution **B** into the another one. Then she took a bottle with some white crystalline powder from the shelf, dissolved it in water (to make approximately 5 percent solution), and added the solution she prepared to each test tube. A thick black precipitate formed in the first test tube, whereas the solution in the second test tube remained unchanged. “Aha”, Alice said. “One more test will clarify everything. Bob, I cannot find a bottle with 5% sodium sulfate solution. Can you please find it and bring it to me?” “Here it is” - Bob replied. “Thank you Bob”, Alice said. She added a small amount of sodium sulfate solution to each of two waste solutions. The solution **A** didn't change, but copious white precipitate formed in the solution **B**. “Look at that, Bob!” Alice said. “Definitely, only a bottle **B** contains heavy metal toxic waste, and the bottle **A** contains just ordinary low toxicity waste. That is a good news, because utilization of low toxicity waste is much less complicated and costly. Please, label the bottles accordingly.”

Explain please, why did Alice come to this conclusion, and which chemical (“a white powder”) did she use? Please, draw the equations of the chemical reactions that took place in these tests.

Good luck!

Hint: To understand what concretely happened when Alice was mixing different solutions, one has to keep in mind that all chemicals Alice was dealing with were *salts*, i.e. ionic compounds that *dissociate* (produce ions) upon dissolution. That means there is no, e.g. $LiNO_3$ molecules in aqueous solution of these salts, there are Li^+ and NO_3^- ions instead. However, when the solutions containing the ions of certain types are mixed together, the ions may stick to each other to form insoluble ionic compounds that form a solid precipitate. The data on solubility of various ionic compounds are summarized in the solubility chart (google it).

BIOLOGY

5 points:

It is known that animals with unusual coloration (albino or melanist) are sometimes born among the normal ones. As a rule, for many reasons, part of which are obvious, this unusual coloration poses a significant disadvantage negatively impacting the animal's survival capabilities. Nevertheless, in some cases unusual coloration may be an advantage.

Please, tell what kind of advantages this abnormality may lead to. Describe as many negative consequences of the animal's unusual coloration as possible.

10 points:

The immune system's primary job is to protect the body from invading pathogens. That can be done because in every organism the outer surface of each cell has unique molecules ("antigens") that serve as a signature of that organism, thereby allowing discrimination between the organism's own cells and alien cells. The mechanism responsible for this discrimination (so called major immune-histocompatibility-complex, or MHC) combines a family of cells that are constantly crawling in between other cells and scanning them for the presence of antigens. The cells containing the organism's own antigens are labeled as "self", whereas the cell containing alien antigens are labelled as "non-self". The cell labelled as "non-self" is attacked and destroyed.

Obviously, since MHS deals with a vast amount of different antigens, its malfunction is possible, when MHC erroneously recognises some of organism's own cells as "non-self", which leads to their destruction. That is a common mechanism of many autoimmune diseases, including diabetes, lupus, rheumatoid arthritis, etc. Interestingly, some autoimmune diseases, such as preeclampsia, develop in healthy women during pregnancy. As a rule, this disease ends after the woman delivers a child.

Given this information:

1. Why are women more vulnerable during pregnancy to autoimmune diseases?

2. If a woman bears two children (A, B), divorces, remarries, and then bears two more children (C, D), during which of her pregnancies is she most likely to develop preeclampsia? Why?

No hints

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in C, C++, Java, or Python
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**

5 points:

A Mad King wonders on a uni-dimensional chess board, with each move going Left (L) or Right (R). Given as an input an initial location of the King (as an integer 1 to 8), and a string of moves, which consists of letters L and R, write a program that determines whether the King returns to its original position at the end of the move sequence and print the result. Note that there is an abyss at both ends of the board, so that the move Left from position 1 and the move Right from position 8 lead to the Mad King falling off the board.

Examples:

Input: initial location: 2, moves: LRRL **Output**: yes

Input: initial location: 2, moves: LLRR **Output**: no

No hints

10 points:

Problem:

A Biosafety Level 4 undercover laboratory is located on a remote island in the Pacific Ocean. It is required by the NSA that a drone continuously patrols the perimeter of the island without ever flying directly over the island. You are to write a program that, given the map of the island, can generate a route for the drone to follow.

The map is provided to you on a square grid. You know that the island is grid-convex, meaning that any horizontal or vertical line on the map is either entirely over sea, or contains just one line segment that is over land. Additionally, one can get from any point on the island to any other point by moving horizontally and vertically only, and the drone can only fly horizontally and vertically also.

The map is provided to you as an array of N strings of length M, where a dot represents water and "#" represents land. Below is an example of a valid island, M = 6, N = 7, with the coordinate system defined

```
(0,0)    (5,0)
   ......
   ......
   ..##..
   .###..
   .#....
   .#....
   ......
(0,6)    (5,6)
```

Two examples of invalid islands (left one is not connected, right one is not convex) are below:

```
                    ......    ......
```

```
....#.    ......
..##..    ..##..
.###..    .###..
.#....    .#....
.#....    .##...
......    ......
```

The drone accepts routing instructions in the form of a sequence of adjacent coordinates on the map. The drone can only fly horizontally and vertically, not diagonally. Your program should thus return a list of coordinates of the squares that the drone is to fly over. **As a reminder, the drone must follow the very perimeter of the island: It must fly over sea only, but every square it flies over must share either a side or a corner with land.**

For example, if the input is:

```
....
.##.
.#..
....
```

The output should be:
00 01 02 03 13 23 22 32 31 30 20 10

No hints