# MATHEMATICS

## 5 points:

Is it possible to arrange all natural numbers from 1 to 42 in a table with 7 columns and 6 rows so that the sum of numbers in each row is the same?

**Hint:** What is the sum of all numbers from 1 to 42?

## 10 points:

Numbers from 1 to 100 are written on the blackboard. In one operation, you can erase any two of the numbers, say, $(a, b)$ and replace the erased pair by the number $a + b + ab$. What number will be left on the blackboard after 99 such operations?

**Hint:** Notice that $a + b + ab = (a + 1)(b + 1) - 1$.

# PHYSICS

## 5 points:

During a rainstorm, an empty bucket moving up with vertical speed V gets filled in 2 minutes. The same bucket moving down with the same vertical speed V gets filled in 8 minutes. How long will it take to fill a motionless bucket?

**Hint:** Let the rain drops fall with some unknown speed U. The rate at which the bucket is filled is proportional to that speed. Therefore, the time to fill the bucket is T=k/U, where k is certain constant. Now consider the case when the bucket goes up and down.

## 10 points:

What horizontal force do you need to apply to this cylinder, mass **M,** to make it roll over the edge?
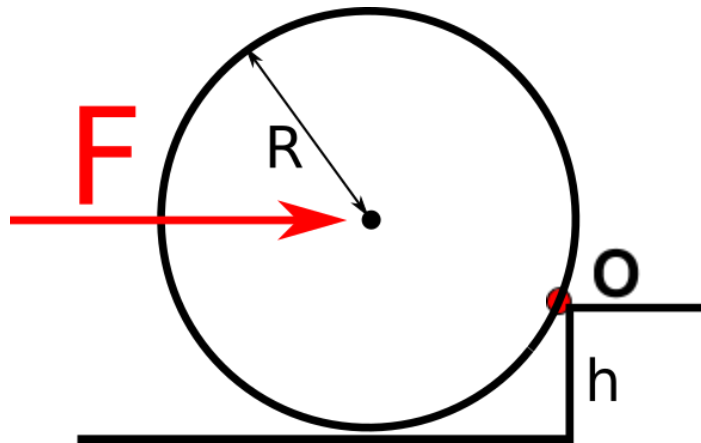


**Hint:** Imagine that the cylinder had just detached from the floor. Sketch all the forces acting on it. In order for it to roll over, the net torque acting on the cylinder, *with respect to the point O*, should be directed clockwise.

# CHEMISTRY

## 5 points:

You probably know that some dyes can change color in acidic or basic media. For example, methyl orange is red in an acidic media (for example, in aqueous acetic acid), becomes yellow in a basic media (for example, in sodium hydroxide solution), and becomes red again if an excess of some acid is added to this yellow basic solution. Such dyes are called *indicators*. Not only they change color in the presence of acids or bases, they do that reversibly. Try to experiment in your kitchen to identify substances that behave like indicators. Of course, it is hardly possible to find a dye that undergo such dramatic color change as methyl orange does, but there should be some materials or food in your kitchen that demonstrate an indicator like behavior.

Do experiments, take pictures showing color changes and send us your pictures with the description of the experiment.

For your experiment, you will need some acid and some base. You can use a lemon juice or vinegar as an acid, but it is not easy to find a reasonably strong base in your kitchen. You can prepare it by decomposing baking soda. Take a teaspoon of baking soda, put it into a cup, add some water (about ⅓ of cup's volume) and microwave for about 20-30 seconds (until the liquid started to boil). That will lead to decomposition of the baking soda according to the equation:

$$2 \text{ NaHCO}_3 \rightarrow \text{Na}_2\text{CO}_3 + \text{H}_2\text{O} + \text{CO}_2$$
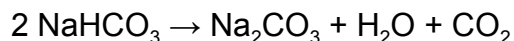
The product of this reaction is washing soda ($Na_2CO_3$ ), and it makes a solution strongly basic, because in water solution washing soda undergoes partial (reversible) exchange with water according to the equation:

$$\text{Na}_2\text{CO}_3 + \text{ H}_2\text{O} \rightleftarrows \text{ NaHCO}_3 + \text{NaOH}$$

(The symbol "$\rightleftarrows$" means the reaction is reversible, so there always is just a small amount of NaOH in the washing soda solution, so it is not dangerous to handle).

Now you know how to obtain both an acid and a base for your experiment. Happy experimenting!

P.S.  When we will be grading this problem, we will pay a special attention to the description of your experiment and the results you obtained. The more convincing your results will be (i.e. the more reliable and more visible the color change is) the more points you get. And, please, don't forget about control: it is highly desirable that you

provide the pictures for the same solution before an acid/base has been added and after that.



## 10 points:

A cargo spacecraft *Giordano Bruno* loaded with various supplies for the new Martian colony was hit by a meteorite on its way to Mars. The damage was relatively minor, but the oxygen tank was completely destroyed, that meant Ridley Scott, the spacecraft's pilot and the only crew member had just few hour to find an new source of oxygen to breathe until the rescue ship arrived (he sent the SOS signal immediately after the accident, but it takes about a week for the rescue spaceship to arrive to *Giordano Bruno*). Upon having inspected his spacecraft's inventory, Ridley found the auxiliary engine's oxidizer tank to contain 38 kg of hydrogen peroxide (the main *Bruno's* engine was powered by a nuclear reactor, so it contained no oxidizer).
In addition to that Ridley found a box with alkaline batteries in the cargo compartment of his ship, as well as several tons of glass wool, which colonists used as a thermal insulator.

**Question #1.** How can Ridley produce oxygen from these materials (explain the most technically feasible way to do that)?

**Question #2.** Will these materials be sufficient to produce enough oxygen for Ridley before the rescue mission's arrival?

To answer the second question, assume the following:
- the only reaction that produces energy in Ridley's body is glucose oxidation (that is a reasonable assumption):

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6CO_2 + 6H_2O$$
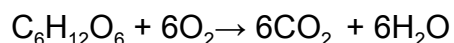
- the heat of combustion of glucose is -2800 kJ/mol  (that means combustion of 180 grams of glucose according to the above equation produces 2800 kJ of energy;180 grams of glucose is 1 mole of glucose; a mole is the amount of any substance measured *in grams* that is numerically equal to its molecular weight in Daltons)
- The minimal amount of energy our body has to produce to support metabolism is 100 kJ per kilogram per day.
- Ridley's weight is 70 kilograms.

# BIOLOGY

**5 points:** There is a raging debate as to whether dogs or cats are more intelligent. Certainly dogs and cats have different types of behavior, but which is more "intelligent" depends upon how we define the term "intelligence." Imagine that you came across an extraterrestrial life form. What specific procedures and criteria would you use to determine which of the life form's behaviors were "hard-wired" versus which behaviors displayed "intelligence?"

**10 points:** DNA consists of two strands (*chromosomes*) lying side-by-side and winding around each other like a spiral staircase. The rungs that connect them are made of two buddies. Each buddy consists of one of four different chemicals (*nucleotides*), nicknamed A, T, G, and C. A gene is nothing but a little segment of the DNA thread containing several thousand nucleotides on one chromosome. The sequence in which the nucleotides are lined up on a gene tells the cell how to make a particular protein. The buddy system means that G is always with C and A is always with T. So if the DNA staircase gets unzipped in the middle and if we had only half of the staircase, we could tell exactly how the other half needs to look, and use new nucleotides (A, T, C, G) to fill in the other half. This property allows our genes not only to divide into two other genes, but also allows them to copy.

Exactly on the spot where a gene begins, the DNA staircase can be opened up a bit. The halves of the rung then become accessible. Now special proteins, called *polymerases*, start doing their job. They always attach a T to an A, an A to a T, a C to a G, and a G to a C. This way a fresh thread of tightly connected nucleotides appears—something called *messenger RNA*. RNA does not contain the nucleotide nicknamed T. Instead, it contains one called U, which takes the place of T. The messenger contains a true copy of the gene—the plan for making a particular protein. The messenger RNA thread leaves the nucleus of the cell with its message. It then goes to the protein factories in the cell, the *ribosomes*, where the messenger RNA's instructions can be carried out. *Now we can understand why different cells can make different proteins (e.g., hair, skin, eyes, muscles, blood) even though the same genes are present in everywhere.* It all depends upon whether or not a messenger RNA copy of a particular gene is made. To switch ON a gene means making a copy of it. For instances, red blood cells would switch ON the gene for hemoglobin by making a

messenger RNA copy of it.  Skin cells would switch OFF the hemoglobin gene by simply not making a copy of it.

While turning genes on and off is necessary for healthy functioning in a body, the process can also go wrong.  A common type of epigenomic modification is called *methylation*. Methylation involves attaching small molecules called methyl groups, each consisting of one carbon atom and three hydrogen atoms, to segments of DNA. When methyl groups are added to a particular gene, that gene is turned off or silenced, and no protein is produced from that gene.  Because errors in the epigenetic process, such as modifying the wrong gene or failing to add a compound to a gene, can lead to abnormal gene activity or inactivity, they can cause genetic disorders. Conditions including cancers, metabolic disorders, and degenerative disorders have all been found to be related to epigenetic errors.

Pick one disease, and explain how errors in turning genes on or off leads to problems.

# COMPUTER SCIENCE

- You can write and compile your code here:
  http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc.
  **No .pdf, .doc, .docx, etc!**

## Introduction:

In computer science, a set is a collection that stores unique items. We already saw 2 collections: array and list. Why do we need another one? Well, sets have some interesting properties:

**1. They guarantee that the items you store in them will be unique, i.e. one of each.** Here's an example:
array: [a, b, c]
list: a → b → c
set: {a, b, c}
If you add 'c' to them, you'll get this:
well, to be precise, let's add to the end of array and list. For sets it's just "add" (we'll discuss later),
[a, b, c, c]
a → b → c → c
{a, b, c} – because we cannot have two 'c'.

**2.** If you want to see if, for example, 'b' is in the collection then for arrays and lists you don't have any choice other than scan the (entire, in worst case) group to see if it's there. For sets it's just a single operation, i.e. it's fast.

**3. Insertion and deletion is fast.**

For example, if you want to add 'd' in the middle of array then you have to:
a. allocate new memory of size n+1, where n is the old size;
b. copy all elements up to where you want to insert d into the new array;
c. copy d;
d. copy all the rest of elements from old to new array;
e. free the memory taken by old array.

For list you have to:
a. start from the beginning of list and traverse the list until you find the spot you want to insert d. This may take some time if the list is large.
b. reassign the "next" link from the previous element to this one ('d');
c. set the "next" link of 'd' element to the next item.

For sets you just insert 'd'. It's kind of 1 operation.

Deletion is similar.

However, sets have a drawback, too. They don't have order. For example, if you start adding letters to a set: {} → {a} → {a, b} → {a, b, c} and then print it, you may see: b, a, c.

## 5 points:
Implement a proof-of-concept spell-checker. Your program should work as follows:
1) Get user input to populate the dictionary
   a) For example, ask user how many words their dictionary has, and then let the user enter those 10 words.
2) Get string from user that needs to be spell-checked.
3) Break string into individual words
4) Check if each word is in the set, and if it is not, let the user know that that word is "misspelled"

## 10 points:
You may have noticed that you didn't need to use sets to implement the spell-checker above. For instance, you could have used arrays or lists to store the dictionary. In this problem, you will investigate whether sets offer speed improvements over the more elementary method.

### Implementing a Microbenchmark test

In this problem we will motivate the use of sets over just arrays or lists. You will create a very large list and a very large set, and you will time how long it takes for the computer to process the same operation with these two methods. Here is the outline of the program you want to write. It really isn't difficult.

Set up:
1) Make an array or list of 10,000 random 10-digit numbers (search online if you're not sure how to do this)
2) Feed this array into a set. [by looping over each list element and running .add() ]

The test to determine how fast lists are:
1) Start a timer. (search google for "how to time things in [python/java..]" if you don't know how to implement a timer)

2) Repeat the following loop N times (make N large enough for the test to take a few seconds):
- Create a random 10-digit number
- Check if the number is in the list by looping over the list manually or using a build-in method for lists (like '.contains()')
3) End timer.

Repeat the same thing for sets. Which one took longer? Is using sets worth it?