PROBLEM OF THE MONTH

January, 2017

2016-2017

MATHEMATICS

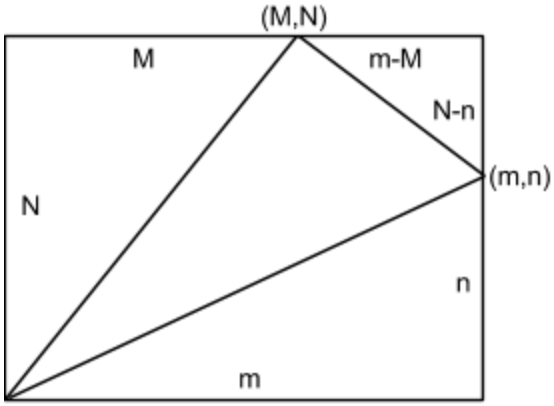
5 points:

The triangle ABC is defined by coordinates of its vertices A=(3,5), B=(10,7), C=(8,11). Find its area.

Hint: Draw this triangle as the triangle inscribed into the rectangle with vertex coordinates (3,5), (10,5), (10,11), (3,11).

Answer: 16

Solution: It is convenient to shift the triangle so that the point A is at the origin (0,0) and points B,C move to the points with coordinates (7,2) and (5,6). To solve even more general problem we consider the triangle with coordinates (0,0), (m,n), (M,N). We assume that m>N>M>n but the formula we obtain will be valid for other cases as well. The triangle is inscribed into the rectangle with vertices (0,0), (m,0), (m,N), (0,N).



We calculate the area of the triangle as follows (see figure):

$A = mN - \frac{1}{2}mn - \frac{1}{2}MN - \frac{1}{2}(m-M)(N-n) = \frac{1}{2}(mN - nM)$.

This is general formula. Substituting m=7, n=2, M=5, N=6 we obtain A=16.

10 points:

The pentagon ABCDE is defined by coordinates of its five vertices on a plane. It is known that all coordinates are integer numbers. Is it possible that the area of the pentagon is equal to 2017/5 ?

Hint: Calculate the area of the triangle with vertices (0,0), (m,n), (M,N). What values can this area take if m,n,M,N are integers?

Answer: Not possible

Solution: Consider triangle with vertices (0,0), (m,n), (M,N). Its area is $\frac{1}{2}(mN - nM)$ (can be derived following the solution of the 5-point problem). It can only take integer and half-integer values if all vertex coordinate are integer. The pentagon can be divided into 5 triangles with all vertices given by integer coordinates. Therefore, the area of the pentagon also can be either integer or half integer and cannot be 2017/5.

PHYSICS

5 pt. Ivar has 50 identical incandescent mini light bulbs, each having constant resistance of 10 Ohms, designed for the current not exceeding 0.35 Amps. He wants to make the brightest possible single-strand Christmas String Lighting Set, powered by the US standard AC 110 Volts outlet. How many bulbs should Ivar use? What will be the value of current in each of the bulbs?

Hint: Find the maximum voltage drop that each bulb may have, and find the minimum number of bulbs connected in series, such that the voltage on each bulb is below that maximum. That will be the optimal string . Why?

Answer: 32 bulbs; $\approx$0.34 Amps.

Solution: The brightest possible single-strand lighting set will be the one that outputs the highest electric power, P=I*V. Since the overall voltage V is fixed at 110 Volts provided by the outlet, the maximum power is achieved for the set which has the highest current, and therefore the smallest resistance. Because resistance adds up when bulbs are connected in series, Ivar needs to connect the smallest possible number of bulbs such that the current through the set is as high as possible while not exceeding the 0.35 Amps limit or, equivalently, the voltage on each bulb does not exceed 0.35Amps*10Ohm = 3.5Volts. For bulbs connected in series the voltage adds up, i.e. the 110 Volts provided by the outlet is equally split between all N bulbs in the lighting set. The smallest N such that 110/N < 3.5 is 32, which means that the current through the set (which is the same for all bulbs) is approximately 110/(10*32)$\approx$0.34 Amps and the electric power is approximately 37.8 Watts.

We note that this solution assumes that in a Christmas Lighting Set all bulbs are equally lit, as it is usually the case. However, if this condition is lifted, an even more powerful lighting set can be designed with 33 bulbs.

10 pt. Ingrid has 350 identical incandescent mini light bulbs, each having constant resistance of 10 Ohms, designed for the current not exceeding 0.35 Amps. She wants

to make the brightest possible multi-strand Christmas String Lighting Set, powered by the US standard AC 110 Volts outlet. How many bulbs should Ingrid use and how should she connect these bulbs? What will be the total power of this set?

Hint: Find the maximum voltage drop that each bulb may have, and find the minimum number of bulbs connected in series, such that the voltage on each bulb is below that maximum. That will be an optimal single string . Why? Now you can take several of these string and connect them in parallel.

Answer: 320 bulbs, $\approx$ 378 Watts.

Solution: The brightest possible lighting set will be the one that outputs the highest electric power, P=I*V. Since the overall voltage V is fixed at 110 Volts provided by the outlet, the maximum power is achieved for the set which has the highest current, and therefore the smallest resistance. For strands connected in parallel the current and power simply add up, so Ingrid has to first optimize each strand to have the highest possible power and current. Because resistance adds up when bulbs are connected in series, for each strand Ingrid needs to connect the smallest number of bulbs such that the current through the strand is as high as possible while not exceeding the 0.35 Amps limit or, equivalently, the voltage on each bulb does not exceed 0.35Amps*10Ohm = 3.5Volts. For bulbs connected in series the voltage adds up, i.e. the 110 Volts provided by the outlet is equally split between all N bulbs in the single strand. The smallest N such that 110/N < 3.5 is 32, which means that the current through the strand (which is the same for all bulbs) is approximately 0.34 Amps. Having 350 bulbs, Ingrid can connect 10 strands in parallel, with the resulting set of approximately 378 Watts with the total current of approximately 3.4 Amps.

We note that this solution assumes that in a Christmas Lighting Set all bulbs are equally lit, as it is usually the case. However, if this condition is lifted, an even more powerful lighting set can be designed with 330 bulbs.

CHEMISTRY

5 points:

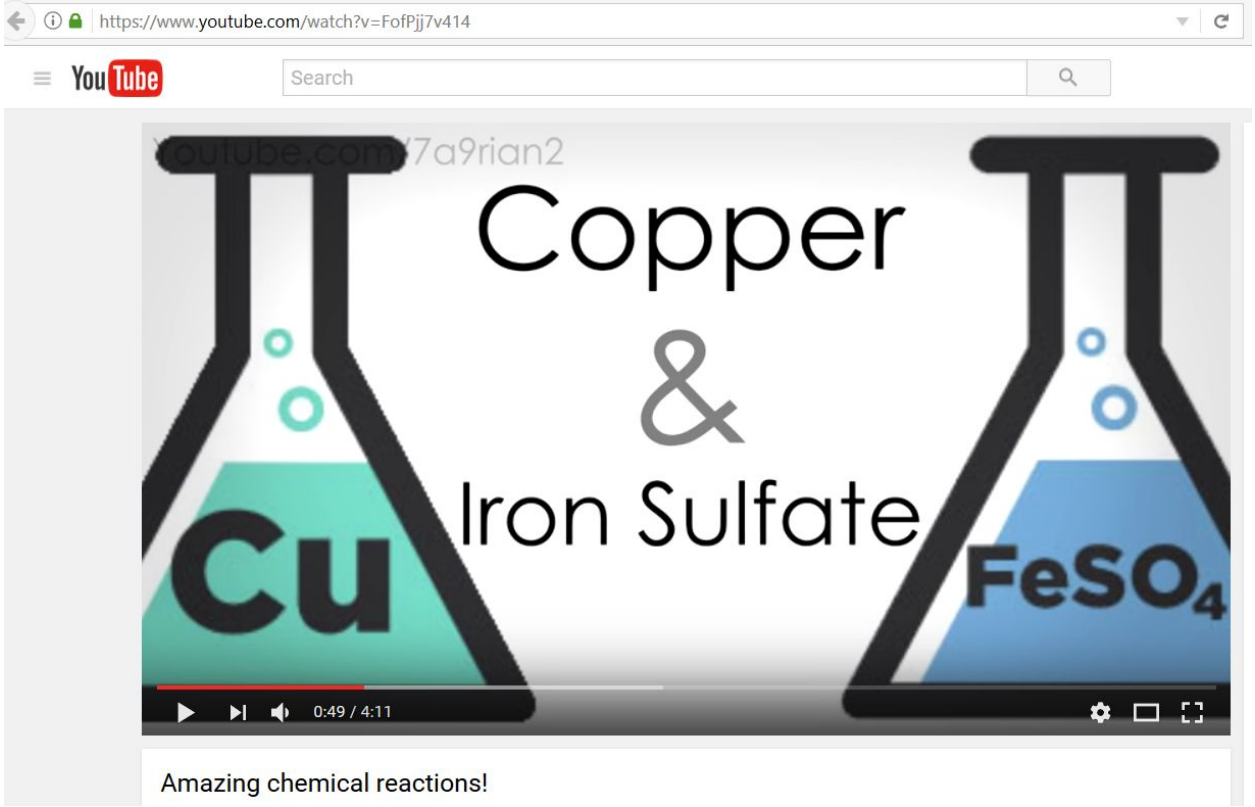
The good thing about YouTube is that everybody can upload their video there. The bad thing about YouTube is … the same. Please, watch this video: https://www.youtube.com/watch?v=FofPjj7v414 (starting from 0:46) and explain what mistake the authors made.

Hint:

This video contains several minor inaccuracies, but there is one absolutely blatant error there.Watch the video starting from 0:40 till 1:05.

Answer:

In addition to several inaccuracies (for example, some “chemical reactions” shown in this video are not chemical reactions at all) there is one absolutely blatant error: the description of the reaction starting after 0:40 says “iron sulfate and copper” (see the picture).



That is absolutely wrong, because copper is right to iron in the metal reactivity series. What does it mean? The metal are arranged in the reactivity series in such a way that

more active metals are in the left side, whereas inert (non-reactive) metals are in the right side. When an active metal (e.g. iron) is in contact with the solution of a salt of a less active metal (e.g. copper) the exchange reaction starts:

$Fe + CuSO_4 \rightarrow Cu + FeSO_4$

In other words, more active metals reduce less active metals from the solutions of their salts. This rule works for other combinations of metals: for example, if you put a copper coin into the solution of mercury nitrate (or other soluble mercury salt) the coin will become silverish due to the formation of a thin film of mercury:

$Cu + Hg(NO_3)_2 \rightarrow Hg + Cu(NO_3)_2$

Similar reaction occurs between copper and silver nitrate, and so on.
In contrast, the reaction between iron sulfate and copper is impossible. By the way, in this video, a dark-reddish sponge precipitates from a green solution. Meanwhile, iron is silverish, not reddish, so this sponge cannot be iron. In addition, the solution of iron (II) sulfate has much less intense color, and it would look colorless at this bright yellow background.
That means, the actual reactants in this reaction were the iron metal and copper sulfate.

In future, please, be careful when you are watching educational materials on Youtube, because sometimes (rarely) they may contain serious errors.

Solution: N/A

10 points:

In the “Taboo” (a British drama TV series, 2016-17) the procedure is described for the detection of the traces of arsenic in biological specimens (in that particular case, it was the content of a stomach of the poisoned man). According to the movie, the specimen is mixed with “potassium oxide, calcium oxide, and nitric acid”. Watch the movie and tell if this method can really detect arsenic, and if not, what should be changed to make the detection of arsenic possible.

Hint:

In this film, the apparatus for arsenic detection and the detection procedure are described correctly, but the chemicals they used are totally weird. Try to tell why it is not possible to convert arsenic in a volatile form using these chemicals?

Answer:

The movie provides a correct description of the apparatus for the Marsh test, the test that allows reliable identification of small traces of arsenic. How does this analytical procedure work?

In this test, the specimen is placed into the flack, and granulated zinc (or zinc powder) and dilute acid are added to it. The flack's neck is connected to the trap (a glass tube filled with some solid that captures water splashes and traces of vapors) and the gas that comes from this trap is ignited. A cold glass or ceramic plate is placed into the fire, and, if the specimen contains arsenic, a black lustrous film ("arsenic mirror") can be clearly seen on this plate.

The chemistry behind that is as follows. When zinc reacts with an acid, hydrogen gas forms according to the equation:

$$Zn + 2\ HNO_3 \rightarrow Zn(NO_3)_2 + H_2$$

At the very first moment, the hydrogen is very active (because the chemical bond in the H_2 molecules has not been formed yet), so it reduces arsenic compounds to the arsenic hydride, or arsine (AsH_3). Arsine is a toxic gas with a very unpleasant odour, but there is no need to smell the reaction mixture, because there is another way to detect it: if we collect all gases that form in this reaction, i.e. all hydrogen and small traces or arsine and burn it, hydrogen will form water:

$$2H_2 + O_2 \rightarrow 2H_2O$$

and arsine will be converted into water and arsenic:

$$2AsH_3 + 3O_2 \rightarrow 3H_2O + 2As$$

The second product, As, is a solid, which will precipitate on a glass or ceramic plate when such a plate is put into the fire.

Why is this method so convenient and sensitive? Because it elegantly resolves the problem of the specimen purification. Biological specimens (especially the content of dead man's stomach) is a very unpleasant thing, with a lot of solids, as well as other materials. Old methods of arsenic detection used some procedure of liquid or solid reduction of an arsenic oxide (which were used as a poison). Arsenic oxide is a colorless

and relatively soluble compound, so it is virtually invisible. In contrast, free arsenic is a black insoluble solid, so if some black solid forms upon treatment of the specimens with reducing agents (hydrogen sulfide, carbon, etc), we can conclude some arsenic compounds were present in these specimens. *However, is it really easy to see a small amount of a black powder in real biological specimens, which are heterogeneous, colored, and contaminated with various, sometimes totally unexpected, stuff?* Or course, no. That is why, old methods of arsenic detection were inconvenient, unreliable, they had low sensitivity and frequently gave false-positive results.
Marsh test was a revolution in arsenic detection, because no other gases besides arsine that form during the treatment of biological specimens with zinc/acid can form a black mirror in these experimental conditions (one exception is antimony, which is toxic per se). That made this test easy to perform, and the sensitivity of arsenic detection was excellent.

What is wrong with the film?
Firstly, the film describes the events that occurred in the very beginning of XIX century, about 20 years before the Marsh test was developed.
Secondly, and most importantly, to produce an arsenic mirror, it is necessary to treat the specimen with some reducing agent, whereas the the chemicals used in this test cannot act as a reducing agent even theoretically, so no arsine can be formed using these chemicals.
Thirdly, the procedure of treatment “with potassium oxide, calcium oxide, and nitric acid” (actually, with potassium carbonate, not oxide), is a procedure for *preliminary treatment* of the specimen which was supposed to be subjected to old and unreliable Metzger’s test. This treatment is not the test by itself.

In summary, although the film correctly describes the apparatus for arsenic detection, this apparatus is an anachronism (it was not known by the moment the events described in the film occurred), and, most importantly, the chemicals used in this test are unsuitable for it.

Solution: N/A

BIOLOGY

5 points:

In deserts it rains only several days a year, usually during "winter" season. How do plants adapt to such conditions? Please give examples.

Answer:

(1) short growing season
(2) hibernate during dry season
(3) store water - cactus
(4) store fat (in addition to starches) which can be used to produce water during seed germination or growth - gourds
(5) long tape root to reach underground water - palm trees
(6) thin leaf cover to reduce evaporation - jade plant

10 points:

Suppose there is a gene variant which encodes for a defective protein. It is known that some people with this gene variant have A serious disorder, caused by the defective protein. Other people with that same variant are seemingly healthy. Please give as many explanations as you can for how this is possible.

Answer:

(1) different genetic background in these people. In seemingly healthy people dysfunctional protein is compensated by presence of other proteins performing the same function.
(2) version of (1) In sick people the gene variant is a "second hit" - the normal version of the gene was compensating for another dysfunctional gene.
(3) in seemingly healthy people the variant which causes disorder is silenced by epigenetic mechanisms.
(4) This gene is only important in certain environmental conditions (like high altitude). Seemingly healthy people are not exposed to this environment.

COMPUTER SCIENCE

- You can write and compile your code here: http://www.tutorialspoint.com/codingground.htm
- Your program should be written in Java or Python
- No GUI should be used in your program: eg., easygui in Python. All problems in POM require only text input and output. GUI usage complicates solution validation, for which we are also using *codingground* site. Solutions with GUI will have points deducted or won't receive any points at all.
- Please make sure that the code compiles and runs on http://www.tutorialspoint.com/codingground.htm before submitting it.
- Any input data specified in the problem should be supplied as user input, not hard-coded into the text of the program.
- Submit the problem in a plain text file, such as .txt, .dat, etc. **No .pdf, .doc, .docx, etc!**

Introduction

We would like to introduce recursion -- a powerful tool to approach algorithm problems.

Example 1
For example, say you want to write a function that computes 10! ("!" stands for factorial, and is defined as 10! = 10 * 9 * 8 * … * 2 * 1). A traditional way is to do that is like this:

```
def factorialSimple(n):
     total = 1
     while n > 1:
          total = total * n
          n = n - 1
     return total
```

Here is how you do it with recursion:

```
def factorial(n):
     if n == 1:
          return 1
     else:
          return n*factorial(n-1)
```

Recursive algorithms are more difficult to understand than regular ones, but can be much more powerful. Let's demonstrate how to understand what a recursive algorithm does.

What happens when you execute command `factorial(1)`? The function returns 1.
What happens if you execute `factorial(2)`? The function will return `2*factorial(1)`, which is of course equal to 2.
What happens if you execute `factorial(3)`? The function will return `3*factorial(2)`, which we already know is the same as `3*2*factorial(1)`, equal to 6. Do you see why this works?

Now, what happens if you execute `factorial(-1)`? The function will return `-1 * factorial(-2)`, where `factorial(-2)` will be evaluated as `-2 * factorial(-3)`, where `factorial(-3)` will be evaluated as `-3 * factorial(-4)`. And so this recursion will continue forever (or, rather, until your compiler/interpreter will gives up and says "`maximum recursion depth exceeded`")

Example 2
Compute x to the power of n:

```
def power(x, n):
    if n == 0:
        return 1
    else:
        return x * power(x, n-1)
```

5 points:

Write a recursive function `bumps(n)` that produces the following output:

```
bumps(1):
.
bumps(2):
.
..
.
bumps(3):
.
..
.
...
.
```

```
..
.
bumps(4):
.
..
.
...
.
..
.
....
.
..
.
...
.
..
.
```

Etc for any positive integer `n`.

Solution:

```
def bumps(n):
      if n == 1:
            print "."
      elif n == 0:
            return
      else:
            bumps(n-1)
            print("."*n)
            bumps(n-1)
```

10 points:

Find the maximum number in an array of integers using recursion.

Solution:

```
def maximum(myarray):
      if len(myarray) == 2:
            if myarray[0] > myarray[1]:
                  return myarray[0]
            else:
                  return myarray[1]
```

```
else:
      maxim = maximum(myarray[1:])
      if myarray[0] > maxim:
            return myarray[0]
      else:
            return maxim
```